Superfluidity in small 2d trapped systems of charged bosons

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Defining the superfluid fraction in confined systems

- Rotating bucket experiment: Only normalfluid component of a liquid responds to slow rotation of the container walls.
- Quantum mechanical moment of inertia $I_{qm}$ deviates from its classical expectation value $I_{class} \rightarrow \textit{non-classical moment of inertia (NCRI)}$

**Definition of superfluid fraction $\gamma_s$**

$$\gamma_s = 1 - \frac{I_{qm}}{I_{class}}$$

Implications of this definition for very small 2$d$-systems containing (2...5) charged particles confined in a harmonic trap.
The path-integral Monte-Carlo method

- Thermal average of an observable $\hat{A}$

$$\langle \hat{A} \rangle = \frac{1}{Z} \int \! dR \int \! dR' \, \rho(R, R'; \beta) \langle R | \hat{A} | R' \rangle$$

- Group property of the density operator

$$\hat{\rho}(\beta) = e^{-\beta \hat{H}} = \left[ e^{-\frac{\beta}{M} \hat{H}} \right]^M = [\hat{\rho}(\beta/M)]^M$$

Discrete time path-integral representation of the density matrix

$$\rho(R, R'; \beta) = \int \! dR_1 dR_2 \ldots dR_{M-1}$$

$$\rho(R, R_1; \tau) \rho(R_1, R_2; \tau) \cdots \rho(R_{M-1}, R'; \tau)$$

(time step $\tau = \beta/M$)
The path-integral Monte-Carlo method

- Each quantum particle is considered as a series of positions forming a closed trajectory in space.
- No spin statistics included → Boltzmannons.
- Symmetrized density matrix

\[
\rho^S(R, R'; \beta) = \frac{1}{N!} \sum_{P \in S_N} \rho(R, PR'; \beta)
\]

- Particle exchange corresponds to larger trajectories including several particles.
- Compute path-integrals by sampling over coordinate and permutation space (Monte-Carlo integration).

M. Bonitz, D. Semkat (eds.): *Introduction to Computational Methods in Many Body Physics*, Rinton 2006
System specification

- Reduced Hamiltonian for a system of $N$ equally charged bosons in a harmonic trap

$$\hat{H}_{\text{red}} = \frac{1}{2} \sum_{i=1}^{N} \left( -\nabla_{x_i}^2 + x_i^2 \right) + \lambda \sum_{i<j}^{N} \frac{1}{|x_i - x_j|}$$

- Reduced units

$$x = \frac{r}{l_0}, \quad \epsilon = \frac{E}{\epsilon_0}, \quad t = \frac{k_B T}{\epsilon_0}, \quad \lambda = \frac{l_0}{a_B},$$

- Harmonic oscillator length $l_0 = \sqrt{\hbar/m\omega}$,
- Energy level spacing $\epsilon_0 = \hbar\omega$,
- Effective Bohr radius $a_B = 4\pi \epsilon_b \epsilon_0 \hbar^2 / m q^2$. 
Density distribution

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Superfluid fraction

Estimator for superfluid fraction $\gamma_s$: area formula

$$\gamma_s = \frac{2m \langle A_z^2 \rangle}{\beta \lambda I_{\text{class}}},$$

- projected area $A = \frac{1}{2} \sum_{i=1}^{N} \sum_{k=0}^{M-1} \mathbf{r}_k^{(i)} \times \mathbf{r}_{k+1}^{(i)}$,
- classical moment of interia
  $$I_{\text{class}} = \left\langle \sum_{i=1}^{N} \sum_{k=0}^{M-1} m_i \mathbf{r}_k^{(i)} \cdot \mathbf{r}_{k+1}^{(i)} \mathbf{r}_k^{(i)} \cdot \mathbf{r}_{k+1}^{(i)} \right\rangle.$$
Superfluidity vs. localization

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Thanks for your attention!
References

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