Surprises in dusty plasmas – attractive forces and magnetization without a magnetic field

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Major contributions to physics of dusty plasmas:

- Pioneer in simulation of dusty plasmas, helped build simulation capabilities at NRL, MPE/Garching, and Kiel.

- Showed linear response theory characterizes interactions between grains in flowing plasma. Used as basis for simulation codes and analytic theory.

- Analytic solution to the Langmuir problem: ion collisions and trapped ions dominate shielding around a dust grain and current flow to the grain.

- Explained high kinetic temperature of dust, due to two-stream instability; freezing into dust crystals is due to frictional stabilization of the instability.

- Used theory and simulation to elucidate the strange nonlinear dynamics of dust grains in flowing plasma: self-propulsion, hysteresis, chaotic oscillations, etc.

- Glenn also contributed to many other areas of plasma physics. He was one of the pioneers of plasma simulation, a leader in electron beam physics and in simulation of plasma processing, and he developed the most important ionosphere simulation code.

Thanks to Martin Lampe
1. Theory of dusty plasmas
2. Attraction of identical particles
3. Magnetized strongly correlated plasmas
4. Magnetizing a dusty plasma without a magnetic field
5. Conclusion and outlook
First principle description of multi-component plasmas

**Complex (Dusty) Plasmas**

- light species: ions, neutrals, electrons (weakly coupled)
- heavy species: dust grains (strongly coupled)

**Problem:** very different masses → r, t – scales
**dust particles:**
polymer micro-spheres (melamine), monodisperse
diameter of a few µm ($d \approx 1...20\mu m$)
highly charged ($Q \approx 10,000e$)
strongly coupled at room temperature ($T \approx 300K$)
low density ($n \approx 1...50mm^{-3}$)
weak to moderate neutral gas friction (gas pressure $p_d \approx 10Pa$)

→ observable with the naked eyes and standard CCD-camera
→ low charge-to-mass-ratio → slow system dynamics
dust plasma frequency: $f \approx (1...10)Hz$
dust particles:
polymer micro-spheres (melamine), monodisperse
diameter of a few µm (d ≈ 10µm)
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weak to moderate neutral gas friction (gas pressure p_d ≈ 10Pa)

→ observable with the naked eyes and standard CCD-camera
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dust plasma frequency: f ≈ (1...10)Hz

→ ideal test system for many-particle correlation effects,
due to the universal character of Coulomb correlations
Laboratory dusty plasmas
„Yukawa balls“: spherical crystals

RF discharge chamber

Experiments
- dust negatively charged (≈10,000e₀)
- gravity compensated by thermophoretic force and electric fields
- glass box avoids formation of void region inside the dust cloud
- confinement (almost) isotropic
- dust particles surrounded by electrons, ions and neutrals

3D Spherical Dust Crystals
strongly coupled Coulomb clusters in traps

Dusty plasma crystal consisting of several tens to thousands dust particles (white dots)

Room temperature, coupling parameter $\Gamma \approx 1000$
3D Spherical Dust Crystals

Parties arranged on concentric spherical shells

Multi-component plasma: electrons, ions, neutral (e.g. argon) atoms, and dust grains

Subsystem of grains (OCP) well described by isotropic Yukawa potential $(Ze)^2 \exp(-\kappa r)/r$

Isotropic 3D parabolic confinement [Arp et al., Phys. Plas. 12, 122102 (2005)]
**Stereoscopy**

*Top:* Scheme of the stereoscopic setup with 3 orthogonal cameras and the expanded laser beam for illumination.  
*Bottom:* Single snap shots of a Yukawa ball with about 60 particles from each camera  

Restricted to small clouds (shadowing effects)


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**Digital Holography**

*Top:* Schematic setup of the stereoscopic digital holography system. The system consists of two identical digital inline holographic setups, with perpendicular orientation of their optical axis.

Stereoscopic holography is suitable for high resolution measurements of 3D dust clouds

Hamiltonian for $N$ identical (classical) dust particles

$$H = H_{\text{conf}} + H_{\text{Yu}} = \sum_{i=1}^{N} \frac{m_i}{2} \omega_0^2 r_i^2 + \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{(Ze)^2 e^{-\kappa |r_i - r_j|}}{|r_i - r_j|}$$

isotropically screened Coulomb interaction: $\kappa \geq 0$

screening independent parabolic confinement


neglect effects of the ion flow (3D dust cloud is in the plasma bulk)

fluctuations negligible (charge, dust grain size)

Hamiltonian for $N$ identical (classical) dust particles

$$H = U_{\text{conf}} + U_{\text{Yu}} = \sum_{i=1}^{N} \frac{m_i}{2} \omega_0^2 r_i^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{(Ze)^2}{|r_i - r_j|} e^{-\kappa \cdot |r_i - r_j|}$$

- isotropically screened Coulomb interaction: $\kappa \geq 0$
- screening independent parabolic confinement
- neglect effects of the ion flow (3D dust cloud is in the plasma bulk)
- fluctuations negligible (charge, dust grain size)

'Molecular' Dynamics

- particle correlations exact
- confinement, screening approximatively
- equilibrium and non-equilibrium
  (real many-particle dynamics)
Hamiltonian for N identical (classical) dust particles

\[ H = U_{\text{conf}} + U_{\text{Yu}} = \sum_{i=1}^{N} \frac{m_i}{2} \omega_0^2 r_i^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{(Ze)^2 e^{-\kappa |r_i - r_j|}}{|r_i - r_j|} \]

\[ \rightarrow \text{Coulomb potential: } \kappa = 0 \]

isotropically screened Coulomb interaction: \( \kappa \geq 0 \)

screening independent parabolic confinement

neglect effects of the ion flow (3D dust cloud is in the plasma bulk)
fluctuations negligible (charge, dust grain size)

'Molecular' Dynamics

particle correlations exact
confinement, screening approximately
equilibrium and non-equilibrium
Spherical Ion Crystals in Traps

- fluorescence pattern of 300, 700, and 1400 laser cooled Ca\textsuperscript{+} ions in a Paul trap (Mortensen et al., University of Aarhus, PRL 2006)
- at temperature $T \approx 5\text{mK}$, coupling parameter $\Gamma = Q^2/\tau k_B T \approx 400$
Concentric spherical shells

**Closed shells (†), „magic“ numbers:**

- $N=12 \ (12,0)$
- $N=13 \ (12,1)$  [Rafac et al. '91]
- $N=57 \ (45,12)$
- $N=58 \ (45,12,1)$
- $N=59 \ (46,12,1)$
- $N=60 \ (48,12)$  [Tsuruta/Ichimaru '93]
- $N=154 \ (98,44,12)$
- $N=155 \ (98,44,12,1)$
- $N=310 \ (165, 94, 41,10)$
- $N=311 \ (166, 89, 43,12,1)$

Earlier results by Rafac et al., Schiffer et al., Hasse/Avilov etc. Often reported metastable states rather than ground states.
Experimental data for N=190
Accurate shell configuration (!): \((107, 60, 21, 2)\)

Theory (MD with \textbf{Coulomb} potential):
ground state configuration: \((115, 56, 18, 1)\)

\(\rightarrow\) Small, but significant differences
Screening of the dust potential

Dust „grain“ immersed in e-i-neutral plasma:

- negatively charged grain surrounded by cloud of positive ions and negative electrons shielding the Coulomb potential
- the grain charge is determined by ion and electron currents $I_{i,e}$

- effective range of the dust-dust pair interaction potential: Debye length

$$\lambda_D^2 = \sum_a \frac{k_B T_a}{e^2 n_a} \quad \kappa = \frac{1}{\lambda_D}$$

Isotropic Yukawa potential

$$\frac{(Ze)^2 e^{-\kappa |r_i - r_j|}}{|r_i - r_j|}$$
N=190

Molecular dynamics with Coulomb and Yukawa-potential

\[ V(r) = \frac{Q^2 e^{-\kappa r}}{4\pi \epsilon_0 r} \]

Experiment vs. Simulation

Experiment

Simulation

\( z-r \)-projection
Experimental data for $N=190$
shell configuration: $(107, 60, 21, 2)$

Theory (MD simulation):
ground state of Coulomb cluster:
$(115, 56, 18, 1)$
Experimental data for $N=190$
shell configuration: $(107, 60, 21, 2)$

Theory (MD simulation):
ground state of Coulomb cluster:
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Yukawa-Potential:
$\kappa r_0$
$0.2$  $114$  $57$  $18$  $1$
Experimental data for N=190 shell configuration: (107, 60, 21, 2)

Theory (MD simulation): ground state of Coulomb cluster: (115, 56, 18, 1)

Yukawa-Potential:

\[ kr_0 = \begin{array}{cccc}
0.2 & 114 & 57 & 18 & 1 \\
0.3 & 111 & 57 & 20 & 2 \\
\end{array} \]
Experimental data for $N=190$
shell configuration: $(107, 60, 21, 2)$

Theory (MD simulation):
ground state of **Coulomb** cluster: 
$(115, 56, 18, 1)$

**Yukawa-Potential:**

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<th>$K r_0$</th>
<th>114</th>
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Yukawa-Potential:

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<td>60</td>
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<td>4</td>
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Exact agreement with experiment!

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Experiment vs. Simulation

N=190

Molecular dynamics with Coulomb and Yukawa-potential
(→ particle correlations exact)

\[ V(r) = \frac{Q^2 e^{-\kappa r}}{4\pi \varepsilon_0 r} \]

Experiment vs. Simulation

(2, 21, 60, 107) \quad (1, 18, 56, 115) \quad (4, 24, 60, 102)

→ shell configuration from MD for \( \kappa=0.6 \): (2, 21, 60, 107) !!
**Experiment (symbols):**
43 clusters, 
N=100...500

Molecular dynamics with Coulomb and Yukawa potential

\[ V(r) = \frac{Q^2 e^{-\kappa r}}{4\pi \varepsilon_0 r} \]

Excellent agreement without free parameters!

*But there is no sensitivity to screening....*
Systematic screening dependence!

→ Non-invasive diagnostics for plasma parameter in experiment:

\[
\kappa = 0.62 \pm 0.23 \\
\kappa = 0.58 \pm 0.43
\]

**Experiment (symbols):**
43 clusters, N=100...500

Molecular dynamics with Coulomb and Yukawa potential

\[
V(r) = \frac{Q^2 e^{-\kappa r}}{4\pi \varepsilon_0 r}
\]

\[
\kappa = \frac{1}{\lambda D} \\
\lambda D^2 = \sum \frac{k B T a}{e^2 n_a}
\]
Systematic screening dependence!

→ Non-invasive diagnostics for plasma parameter in experiment:

outer (1th) shell: \( \kappa = 0.62 \pm 0.23 \)

2nd shell: \( \kappa = 0.58 \pm 0.43 \)

Experiment (symbols):
43 clusters, \( N=100...500 \)

Molecular dynamics with Coulomb and Yukawa potential

\[
V(r) = \frac{Q^2 e^{-\kappa r}}{4\pi \varepsilon_0 r}
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\[
\kappa = \frac{1}{\lambda_D}
\]

\[
\lambda_D^2 = \sum_a \frac{k B T a}{e^2 n_a}
\]

→ excellent agreement with dusty plasma experiments (Phys. Rev. Lett. 2006)
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Experiments with fast streaming ions

String Formation in Vertically Elongated 3D Confined Dusty Plasmas
Representative examples for various plasma parameters


N. Sato et al. (2001)

A. Schella, A. Melzer et al. (2011)

Vertical alignment not explainable with repulsive Yukawa potential!
Effect of fast streaming ions

Scheme of the experimental setup

Close to electrode: strong field, fast ions → supersonic motion, Mach cone

Early work:
Schweigert et al.
Morfill et al.
Piel, Melzer
Joyce, Lampe
Effect of fast streaming ions

Scheme of the experimental setup

Close to electrode: strong field, fast ions
→ supersonic motion, Mach cone

Effective dust potential not isotropic
Different from Debye/Yukawa
→ non-equilibrium effect

Streaming plasma
→ wake-field behind charged grain
→ non-reciprocal grain interaction
→ vertical grain alignment

Figure: J. Carstensen
Grain potential in a streaming plasma

No drift

Electric field-induced ion drift

Debye sphere

Ion wake* in the center of the asymmetric cloud

Isotropic Yukawa (Debye) Potential:

$$\phi = \frac{(Ze)^2 e^{-\kappa \cdot |r_i - r_j|}}{|r_i - r_j|}$$

$$\kappa = \frac{1}{\lambda_D}$$

$$\lambda_D^2 = \sum_a \frac{k_B T_a}{e^2 n_a}$$

Fig.: Ivlev et al, PRL 100, 095003 (2008)
Other wake fields:

- Surfing
- Laser wake field acceleration
- Electrons in undulator (FEL)
...
Nonequilibrium theory:
Solve coupled kinetic and Maxwell’ equations

\[
\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0,
\]

\[
- \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \nabla \times \mathbf{B} = \mu_0 \mathbf{J},
\]

\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0,
\]

\[
\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0},
\]

\[
\nabla \cdot \mathbf{B} = 0.
\]

with

\[
\rho(x, t) = \sum_s q_s \int f_s(x, \mathbf{v}, t) \, d\mathbf{v}, \quad \mathbf{J}(x, t) = \sum_s q_s \int f_s(x, \mathbf{v}, t) \mathbf{v} \, d\mathbf{v}
\]

f\_s distribution function of electrons, ions, neutrals, dust
PIC simulations: The Vlasov-Maxwell system

- The one species Vlasov equation reads
  \[
  \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_x f + \mathbf{E} \cdot \nabla_v f = 0.
  \]

- It is coupled to the Poisson equation
  \[
  -\Delta \phi = 1 - \rho = 1 - \int f(t, x, v) \, dv, \quad E = -\nabla \phi.
  \]
Scheme of the Particle-in-Cell algorithm

- Assign charges to grid points
- Calculate electric field at grid points (Poisson eq.)
- Weight field to particle positions (calculate forces)
- Advance particles (equation of motion) new velocities and positions
- Check for boundaries: remove/add particles
- MC: Check for collisions, add/remove particles

Efficient solution of Vlasov-Maxwell system: Particle-in-Cell (PIC) Simulations

- includes nonlinear effects
- grain charging included


**Problem so far**: selfconsistent treatment of the heavy particles

- only very few grains: N=2-3
- no dust dynamics

Mass ratio electron/ion/dust: 1 / 40*1850 / 10^{17}

Ratio of characteristic time scales (inverse plasma frequency):
1 / 300 / 10^{8}

→ selfconsistent simulation of dust, electron and ion dynamics impossible (and not needed)
→ solution: multi-scale approach
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Dynamical screening (linear response) approach


- Linearized Vlasov-Poisson equation around Maxwellian equilibrium reads

\[
\frac{\partial f^1}{\partial t} + v \frac{\partial f^1}{\partial x} - \frac{e}{m} E^1(x) \frac{df^0}{dv} = 0,
\]

\[
\frac{dE^1}{dx} = -\frac{e}{\epsilon_0} \int_{-\infty}^{+\infty} f^1(x, v, t) \, dv,
\]

with initial condition \( f^1(x, v, 0) = f^1_0(x, v) \).

→ Compute dielectric function of electrons and ions
Dynamically screened potential

\[ \Phi_i(r, t) = \int \frac{d^3k}{2\pi^2} \frac{q}{k^2} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{v}_i t)} \varepsilon^1(\mathbf{k}, \mathbf{k} \cdot \mathbf{v}_i) \]

Dust grains are „dressed“, mediated by dielectric function

Fourier transform of bare Coulomb potential
Dynamically screened potential

\[ \Phi_i(\mathbf{r}, t) = \int \frac{d^3 k}{2\pi^2} \frac{q}{k^2} \frac{e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{v}_i t)}}{\epsilon^1(\mathbf{k}, \mathbf{k} \cdot \mathbf{v}_i)} \]

Dust grains are „dressed“, mediated by dielectric function

Dielectric function for a (shifted) Maxwellian plasma with BGK-type collisions included

\[ \epsilon^l(\mathbf{k}, \omega) = 1 + \frac{1}{k^2 \chi^2_{De}} + \frac{1}{k^2 \chi^2_{Di}} \left[ 1 + \frac{\zeta_i Z(\zeta_i)}{1 + \frac{i\nu_{in}}{\sqrt{2kT_i}} Z(\zeta_i)} \right] \]

electrons: statical screening \((u_e << V_{Te})\)
ions: dynamical screening \((T_i < T_e)\)

\[ \chi^2_{D\alpha} = \frac{\nu_p^2}{\omega_p^2} = \frac{\varepsilon_0 k_B T_{\alpha}}{n_{\alpha} q_{\alpha}^2} \]

Ion-neutral scattering → collisional damping

bar Coulomb potential
static screening → Yukawa potential
dynamical screening → wake effects

\[ \zeta_i = \frac{\mathbf{k} (\mathbf{v}_d - \mathbf{u}_i) + i\nu_{in}}{\sqrt{2kT_i}} \]  

ion neutral collision frequency

thermal velocity \(v_{T\alpha} = \sqrt{\frac{k_B T_{\alpha}}{m_{\alpha}}}\)

Plasma dispersion function:

\[ Z(z) = i\sqrt{\pi} w(z) \]

\[ w(z) = \exp \left( -z^2 \right) \text{Erfc}(iz) \]

\[ = \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-t^2)}{z-t} dt \]

No ion flow: Yukawa (Debye) Potential

Statically screened Coulomb (Yukawa) potential

\[
\frac{(Ze)^2 e^{-\kappa |r_i - r_j|}}{|r_i - r_j|} \Rightarrow \kappa = \frac{1}{\lambda_D} \\
\lambda_D^2 = \sum_a \frac{k_B T_a}{e^2 n_a}
\]

Plasma parameters (Argon): n=2x10^8 cm^-3, T_e=2.585eV (30000K), T_e/T_i=30, v_in=0.1\omega_{pi}, \lambda_{De}=0.845mm
Potential for Streaming Ions: $M=0.250$

Dynamically screened Coulomb potential: 'wake' potential (first peak height: 10.9mV @ $z=0.29\text{mm}$)

Mach number $M \equiv \frac{u_i}{c_s}$, Bohm speed $c_s \equiv \sqrt{\frac{k_B T_e}{m_i}}$

Plasma parameters (Argon): $n=2\times10^8\text{cm}^{-3}$, $T_e=2.585\text{eV (30000K)}$, $T_e/T_i=30$, $u_{in}=0.1\omega_p$, $\lambda_D=0.845\text{mm}$

Potential for Streaming Ions: \( M = 0.375 \)

Dynamically screened Coulomb potential: 'wake' potential (first peak height: 21.7mV @ \( z = 0.39 \)mm)

Mach number \( M \equiv \frac{u_i}{c_s} \), Bohm speed \( c_s \equiv \sqrt{\frac{k_B T_e}{m_i}} \)

Plasma parameters (Argon): \( n = 2 \times 10^8 \text{cm}^{-3}, T_e = 2.585 \text{eV (30 000K)}, T_e/T_i = 30, u_{in} = 0.1 \omega_{pi}, \lambda_{De} = 0.845 \text{mm} \)

Dynamically screened Coulomb potential: 'wake' potential
(first peak height: 31.2mV @ z=0.46mm)

Mach number $M \equiv \frac{u_i}{c_s}$, Bohm speed $c_s \equiv \sqrt{\frac{k_BT_e}{m_i}}$

P.Ludwig, W.J. Miloch, H. Kählert, M. Bonitz,

Plasma parameters (Argon): $n=2\times10^8\text{cm}^{-3}$, $T_e=2.585\text{eV (30000K)}$, $T_e/T_i=30$, $u_{in}=0.1\omega_{pi}$, $\lambda_{De}=0.845\text{mm}$
Potential for Streaming Ions: $M=0.6875$

Dynamically screened Coulomb potential: 'wake' potential (first peak height: 35.3mV @ $z=0.64\text{mm}$)

Mach number $M \equiv \frac{u_i}{c_s}$, Bohm speed $c_s \equiv \sqrt{\frac{k_BT_e}{m_i}}$

Plasma parameters (Argon): $n=2\times10^8\text{cm}^{-3}$, $T_e=2.585\text{eV (30000K)}$, $T_e/T_i=30$, $u_{in}=0.1\omega_{pi}$, $\lambda_{De}=0.845\text{mm}$

Potential for Streaming Ions: M=1.00

Dynamically screened Coulomb potential: 'wake' potential (first peak height: 29.9mV @ z=0.97mm)

Mach number $M \equiv \frac{u_i}{c_s}$, Bohm speed $c_s \equiv \sqrt{\frac{k_B T_e}{m_i}}$

Plasma parameters (Argon): $n=2\times10^8 \text{ cm}^{-3}$, $T_e=2.585\text{eV (300 000K)}$, $T_e/T_i=30$, $u_{in}=0.1\omega_{pi}$, $\lambda_{De}=0.845\text{mm}$

Linear Dielectric Response ansatz validated against full nonlinear 3D Particle-in-Cell (PIC) simulations

3D PIC by W. Miloch (top) vs LR results (bottom) for $M=1.5$, $T_e/T_i=10$, and collision freq. $\nu_{in}=0$.

LR results: Peak positions of the wake potential as function of Mach number $M$. Red (blue) lines correspond to a positive (negative) space charge.

Diamonds: 3D PIC results for the wake maxima (collisionless plasma, W. Miloch)

Potential cuts through the grain along the flow direction

LR: finite damping included $v_{in}=0.1$
COPTIC: collisionless PIC*
Lampe et al.: collisionless LR

PIC (COPTIC) results* for different collision frequencies vs. Linear Response calculation for $v_{in}=0.1$.

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* I. Hutchinson, Phys. Plasmas 18, 032111 (2011)
Linear Dielectric Response ansatz validated against full nonlinear 3D Particle-in-Cell (PIC) simulations

Potential cuts through the grain along the flow direction

LR: finite damping included $v_{in}=0.1$, COPTIC: collisionless PIC* Lampe et al.: collisionless LR

**PIC (COPTIC) results** for different collision frequencies vs. Linear Response calculation for $v_{in}=0.1$.

Excellent quality of the linear LR results $\rightarrow$ accurate N-particle-dynamics

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* I. Hutchinson, Phys. Plasmas 18, 032111 (2011)
Hamiltonian of the e-i-n-particle plasma

\[ H = \sum_{a=e,i,n,d}^{N_a} \sum_{i=1}^{N_a} h_i^{(a)}(E, B) + W_{dd} + W_{de} + W_{di} + W_{dn} + W_{e+i+n}^{\text{plasma}} \]

**Approximations:**

1. neutrals in thermal equilibrium

2. average over particle (d) plasma period → stationary e-i flow

3. Fokker-Planck approximation for d-n interaction (Langevin dynamics)

4. weak d-i and d-e coupling → linear response for e-i dynamics

5. fixed dust charge

N-particle simulation of a realistic plasma
Multiscale approach

Langevin dynamics scheme for correlated dust:

\[ m_d \ddot{r}_k = -\nabla V_k^{\text{eff}}(r, t) - \omega_0^2 m_d r_k - \nu_{dn} m_d \dot{r}_k + \mathbf{f}_k(t) \]

\[ V_k^{\text{eff}}(r, t) = \sum_{l \neq k}^{N_d} q_d \Phi_l(r, t) \quad \Phi_i(r, t) = \int \frac{d^3 k}{2\pi^2} \frac{q}{k^2} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{v}_i t)} \]

friction coefficient, Gaussian random force, and plasma temperature are related by the fluctuation-dissipation theorem

\[ \langle f_i^\alpha(t) f_j^\beta(t') \rangle = 2m \nu_{dn} k_B T \delta_{ij} \delta_{\alpha\beta} \delta(t - t'), \quad \alpha, \beta \in \{x, y, z\} \]

Approximations:
1. neutrals in thermal equilibrium
2. average over particle (d) plasma period → stationary e-i flow
3. Fokker-Planck approximation for d-n interaction
4. weak d-i and d-e coupling → lineare response for e-i dynamics
5. fixed dust charge
Multiscale Dust Dynamics Simulations
Snapshots, 46 particles in a trap

Simulation parameters

Electrons: $n_e = n_i = 1.0 \times 10^{14}/m^3$, $T_e = 2.5$eV, $\lambda_{De}(T_e, n_e) = 1175.41 \mu m$

Ions (Argon, $Z=1$): temperature: $T_e/T_i = 83.333$ ($T_i = 0.03$eV = 348K), $\lambda_{Di}(T_i, n_i) = 128.76 \mu m$

Neutrals: gas pressure: 15Pa, scattering freq.: $\nu_{in}/\omega_i = 0.201$, $T_n = T_i$

Dust: charge: $Q_d = -6000e_0$, coll. freq.: $\nu_{dn} = 19.08$Hz, radius: $R_d = 2.43 \mu m$, $m_d = 9.1 \times 10^{-14} kg$

trap: $\omega_0 = \omega_z = 7.0$Hz, $c_s = \sqrt{(k_B T_e/m_i)} = 2460$m/s, $\lambda_{D,tot} = \lambda_{De}^{-2} + \lambda_{Di}^{-2} = 128.00 \mu m$, $\lambda_{D\alpha} = \sqrt{(\varepsilon_0 k_B T_\alpha / (n_\alpha q_\alpha^2))}$

Multiscale Dust Dynamics Simulations

Streaming induced phase transition. Snapshots, 46 particles in a trap

P. Ludwig et al., Ion-Streaming Induced Order Transition in 3D Plasma Crystals, PPCF 54, 045011 (2012)
Multiscale Dust Dynamics Simulations
First Principle Simulations of Strongly Correlated Dusty Plasma

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Particle Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_z = 0.5\omega_0 = 7.5\text{Hz}$</td>
<td>$N=32$</td>
</tr>
<tr>
<td>$\omega_z = \omega_0 = 15\text{Hz}$</td>
<td></td>
</tr>
<tr>
<td>$\omega_z = 1.5\omega_0 = 22.5\text{Hz}$</td>
<td>$N=100$</td>
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<tr>
<td>$\omega_z = 2.0\omega_0 = 30\text{Hz}$</td>
<td>$N=100$</td>
</tr>
</tbody>
</table>

Simulation snapshots
Multiscale Dust Dynamics Simulations

Videos

M=0.01, 46 particles side view

M=1.0, 46 particles side view

M=1.33, 46 particles topview

Simulations: Patrick Ludwig
Conclusions (1)

1. Highly charged particles in plasma produce strongly correlated states

2. Accurate experimental diagnostics on single-particle level → insight into structure, dynamics (spectrum), phase transitions in confined systems
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4. Open system, nonequilibrium, streaming ions:
   → Attractive dust-dust interaction, wake potentials

http://www.theo-physik.uni-kiel.de/~bonitz

Bonitz et al., Reports Prog. Phys. 73, 066501 (2010)
Bonitz et al. (eds.), „Introduction to Complex Plasmas“, Springer 2010