Multiple undamped acoustic plasmons in three-dimensional two-component nonequilibrium plasmas

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Within the random-phase approximation, we demonstrate the existence of multiple undamped acoustic plasmons in a two-component three-dimensional plasma with an isotropic momentum distribution.

I. INTRODUCTION

The collective modes of electron-hole plasmas in excited semiconductors play an important role in many-body processes such as carrier-carrier scattering, screening of the Coulomb interaction potential, and optical dephasing. These plasma excitations are well understood for equilibrium and near-equilibrium conditions where the so-called optical plasmon is essentially the only undamped mode. Recently we discussed the possibility of an undamped acoustic mode that can exist (along with an optical mode) for a highly nonequilibrium distribution with unoccupied low-momentum states (Refs. 1 and 2). Such distributions can be created, for example, by intense femtosecond optical pulses with a center frequency well above the band edge.

Generally, plasmas with distributions that are nonmonotonic as functions of wave number can exhibit instabilities. These instabilities can be viewed in terms of energy transfer between collective excitations and the single-particle excitations. If there are more particles with velocities just above the phase velocity of the plasmon than just below, then, under certain conditions, the plasmon grows because more energy is gained from the faster particles than is lost to the slower ones.

In this paper we present results along the lines of Ref. 2, where we studied dielectric properties of plasmas on the basis of the random-phase approximation (RPA). We show that there are four undamped collective modes for a two-component three-dimensional isotropic plasma with a specific distribution consisting of two nonequilibrium boxes. Three of these modes can be called acoustic (in other words they have vanishing frequency at zero momentum) and one is optical. We also present a plot of the real versus imaginary part of the longitudinal dielectric function (i.e., a Nyquist diagram, Ref. 3) showing that these modes are stable (within the RPA theory).

II. THEORETICAL DESCRIPTION OF LONGITUDINAL PLASMA OSCILLATIONS

For our calculations, we deal with the retarded longitudinal polarization function \( P(q, \omega) \) and the dielectric (or screening) function \( \epsilon(q, \omega) = 1 - V_q P(q, \omega) \). Here, \( V_q \) is the unscreened Coulomb potential given by

\[
V_q = \frac{4\pi e_0^2}{\epsilon_0 q^2 L^3},
\]

where \( e_0 \) is the free electron charge, \( \epsilon_0 \) is the background dielectric constant, and \( L^3 \) is the volume. Within the random-phase approximation (RPA) the polarization is given by

\[
P_\alpha(q, \omega) = \lim_{\delta \to 0} \frac{2}{\hbar \omega + i\delta + E_\alpha(k) - E_\alpha(|k + q|)},
\]

where \( \alpha = e \) or \( h \) and \( E_\alpha(k) \) is the single particle kinetic energy given by \( E_\alpha(k) = \hbar^2 k^2/2m_\alpha \). The effective particle mass, which is defined by the curvature of the band dispersion, is used for \( m_\alpha \) (for our calculations, \( m_e = 1.284 \) and \( m_h = 4.522 \) in units of the reduced mass). The total polarization is the sum of the electron and hole polarization (i.e., \( P = P_e + P_h \)). Because we limit our discussion to isotropic distributions, all functions depend only on the magnitude of \( k \).

If we approximate optically generated distribution functions by boxlike functions [e.g., \( f(k) = A \Theta(k_2 - k) \Theta(k - k_1) \)], the integration in Eq. (2) is easily performed to yield

\[
P_\alpha(q, \omega) = \tilde{P}(q, -\hbar v_\alpha, \epsilon_\alpha^+(q)) + \tilde{P}(q, \hbar v_\alpha, \epsilon_\alpha^-(q)) + \tilde{P}(q, \hbar v_\alpha, \epsilon_\alpha^+(q)) + \tilde{P}(q, -\hbar v_\alpha, \epsilon_\alpha^-(q)),
\]

where

\[
\tilde{P}(q, a, b) = \frac{m_\alpha A L^3}{4\pi^2 \hbar^2 q} \left\{ k_2^2 \ln(ak_2 + b) - k_1^2 \ln(ak_1 + b) - \frac{1}{2} \left[ k_2^2 - k_1^2 \right] - \frac{1}{a^2} \left[ ab(k_1 - k_2) + b^2 \ln\left( \frac{ak_2 + b}{ak_1 + b} \right) \right] \right\},
\]

\[
v_\alpha \equiv \frac{\hbar q}{m_\alpha},
\]

\[
\epsilon_\alpha^\pm(q) \equiv \hbar \omega \pm \frac{\hbar^2 q^2}{2m_\alpha} + i\delta.
\]
For numerical reasons δ is chosen to be very small rather than zero (for these calculations δ = 0.000 005ω_p1, where ω_p1 is the plasma frequency). Further discussion and physical interpretation of the RPA polarization in nonequilibrium situations can be found in our previous paper (Ref. 2).

III. DYNAMICAL STRUCTURE FACTOR FOR ONE- AND TWO-COMPONENT PLASMAS

In Ref. 2, we demonstrated the interesting result that it is possible to have two undamped collective modes in a one-component plasma for a suitable distribution (in this case one consisting of a zero temperature part plus a separate higher-momentum “box”). For a two-component plasma using this same distribution for both components, we found that there still were only two undamped collective modes. This occurs because the undamped hole modes (modes of the heavier species) from the single component plasma occur within the pair continuum of the electrons (the lighter species). Therefore these two modes are damped in the two-component plasma.

By choosing a distribution consisting of two boxes (like that above), but with neither box occupying the low k states, it is possible for these modes to be undamped in the two-component plasma. In Fig. 1 we show plots of the dynamical structure factor, \( S(q, \omega) = -\text{Im}(\epsilon^{-1}(q, \omega))/nV_q \) (where n is the plasma density), which demonstrate this situation. The distribution function used for both electrons and holes is \( (k \text{ is in units of } a_{\text{Bohr}}^{-1}) \)

\[
\begin{align*}
  f(k) &= 0.2 \Theta(3.5 - k) \Theta(k - 2.8) \\
  &+0.2 \Theta(5.71 - k) \Theta(k - 5.5).
\end{align*}
\]  

(7)

The structure factor clearly exhibits two undamped modes for both the electron [Fig. 1(a)] and hole [Fig. 1(b)] single-component plasmas. The difference occurs when the electrons and holes are combined to produce a two-component plasma [Fig. 1(c)] which now has four undamped collective modes (three acoustic and one optical). In fact, it is possible to produce an arbitrary number of undamped acoustic modes by simply adding enough carefully-chosen peaked-structures to the distribution functions.

IV. STABILITY OF COLLECTIVE MODES

The stability of collective modes in classical plasmas (including those with model distributions like the ones used in this paper) has been extensively investigated (Refs. 3 and 4). In a classical isotropic three-dimensional plasma all collective modes have been shown to be stable against small external fluctuations (Ref. 4, p. 113). While, in general, the results for classical plasmas cannot be extended to quantum systems, we would still expect the collective modes found in our calculation to be stable.

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**FIG. 1.** Density plots showing the logarithm of the dynamical structure factor for (a) a single component electron plasma, (b) a single component hole plasma, and (c) a two-component electron-hole plasma. Darker regions correspond to higher values. The frequency ω is plotted in units of the exciton Rydberg energy \( E_R \) and the wave number q is plotted in inverse Bohr radii \( a_{\text{Bohr}}^{-1} \) (\( E_R = 4.2 \text{ meV for these calculations} \)).

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**FIG. 2.** (a) Nyquist diagram for two-component electron-hole plasma. In (b) we zoom in on the origin to illustrate that it is not enclosed by the curve.
In order to verify the stability of the modes we plot in Fig. 2 the imaginary part of the dielectric function versus the real part, for \( q = 0.4a_{\text{Bohr}}^{-1} \), letting \( \omega \) vary between positive and negative infinity (\( a_{\text{Bohr}} \) is the exciton Bohr radius, taken to be 140 Å for these calculations). This type of plot is known as a Nyquist diagram, and is applicable to both quantum and classical plasmas (Refs. 3, 5, and 6). Since \( \epsilon(q, \omega \to \infty) \to 1 \) a closed curve is formed. If there is an unstable mode then we expect that the origin (\( \epsilon = 0 \)) will be enclosed within the curve. We notice from Fig. 2(b) that the origin is not enclosed by any loop, indicating that there are no unstable modes for this situation.

V. CONCLUSIONS

In this paper we have built on ideas presented in our previous paper (Ref. 2) to show the existence of multiple undamped acoustic modes in a two-component quantum plasma. Using plots of the dynamical structure factor we demonstrate the existence of three undamped acoustic modes and one undamped optical mode for a two-component plasma whose distribution function consists of separate “boxes.” In principle, it would be possible to generate an arbitrary number of undamped acoustic modes by using a suitable distribution function. While for simplicity of analysis we limit our calculations to distributions consisting of boxes, these results are not artifacts of the distribution’s discrete nature and qualitatively similar results can be expected if the boxes are replaced with smooth, sufficiently separated Gaussians (for example). It should be noted that these calculations are done using the random-phase approximation and, as such, are probably valid only for weak external disturbances in the high density limit. Further, the effects of collisions are excluded from the calculation and would tend to broaden the collective modes and pair continua borders.

While difficult, experimental observation of the multiple acoustic modes should be possible. Similar, double-peaked distributions can be generated in a semiconductor by optical excitation (e.g., for a system which has two simultaneously dipole coupled valence bands, such as the heavy- and light-hole bands in GaAs). Using a backscattering technique similar to that used in Ref. 7, the collective modes may be observable. Further, the undamped plasmons, through their influence on the screening, can lead to ultrafast carrier scattering rates (as shown in Ref. 1 as a single lightly-damped acoustic mode).

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